ECTRI – FEHRL – FERSI
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TRANSPORT USER BENEFITS MEASURE FOR TRAVEL DEMAND MODELS WITH CONSTRAINTS

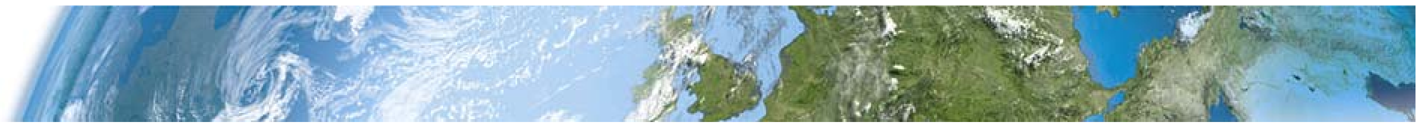
Christian Winkler
Institute of Transport Research
German Aerospace Center

DLR

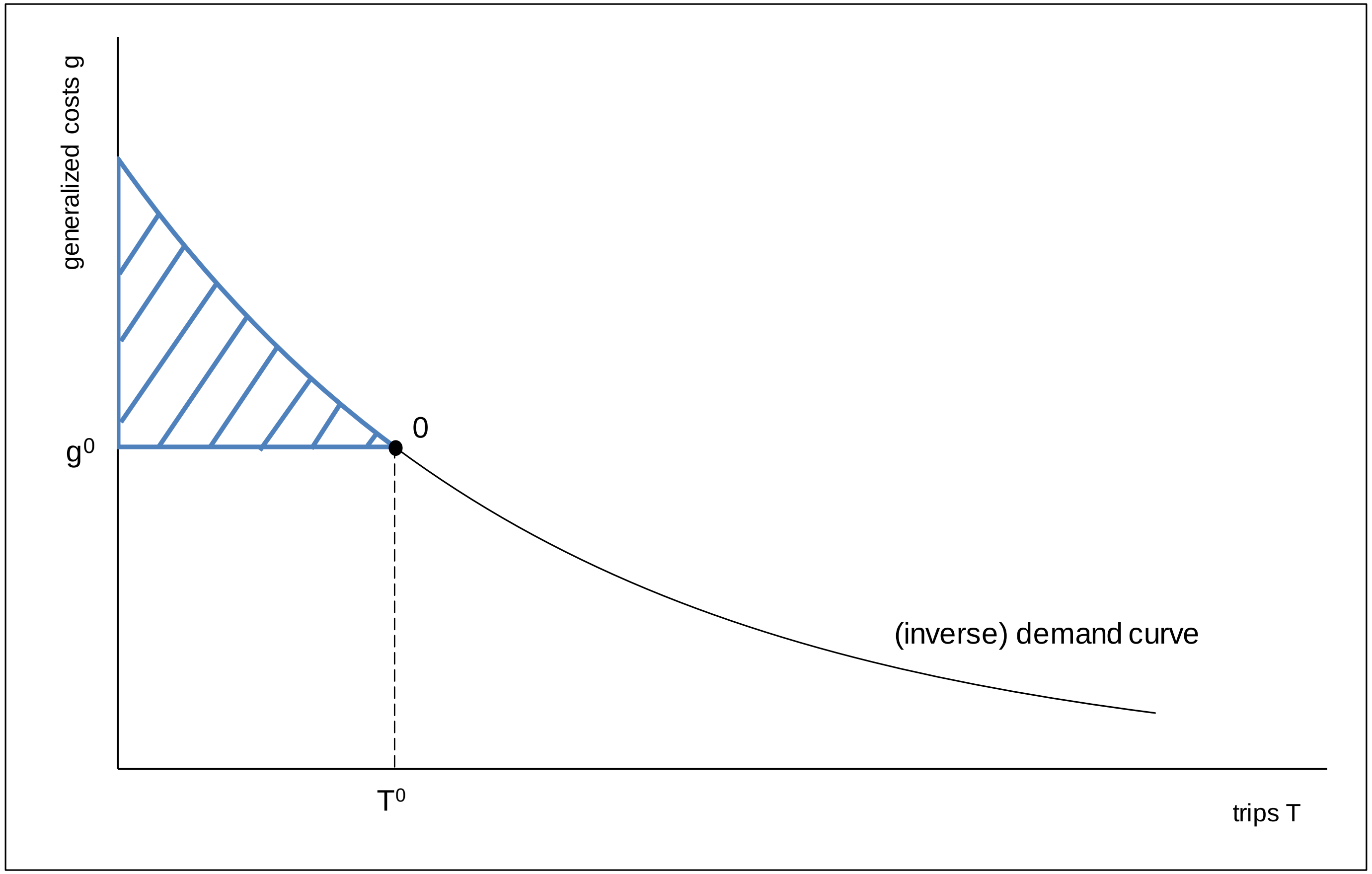
Overview

- Transport investments are often associated with high investment costs for society and cost changes for transport users
- Anticipation of welfare impacts is essential
- Decision makers need robust and easy decision advice
- Specific guidelines and rules are implemented in most countries
- Cost benefit analysis is the standard approach
- The basic calculation is:

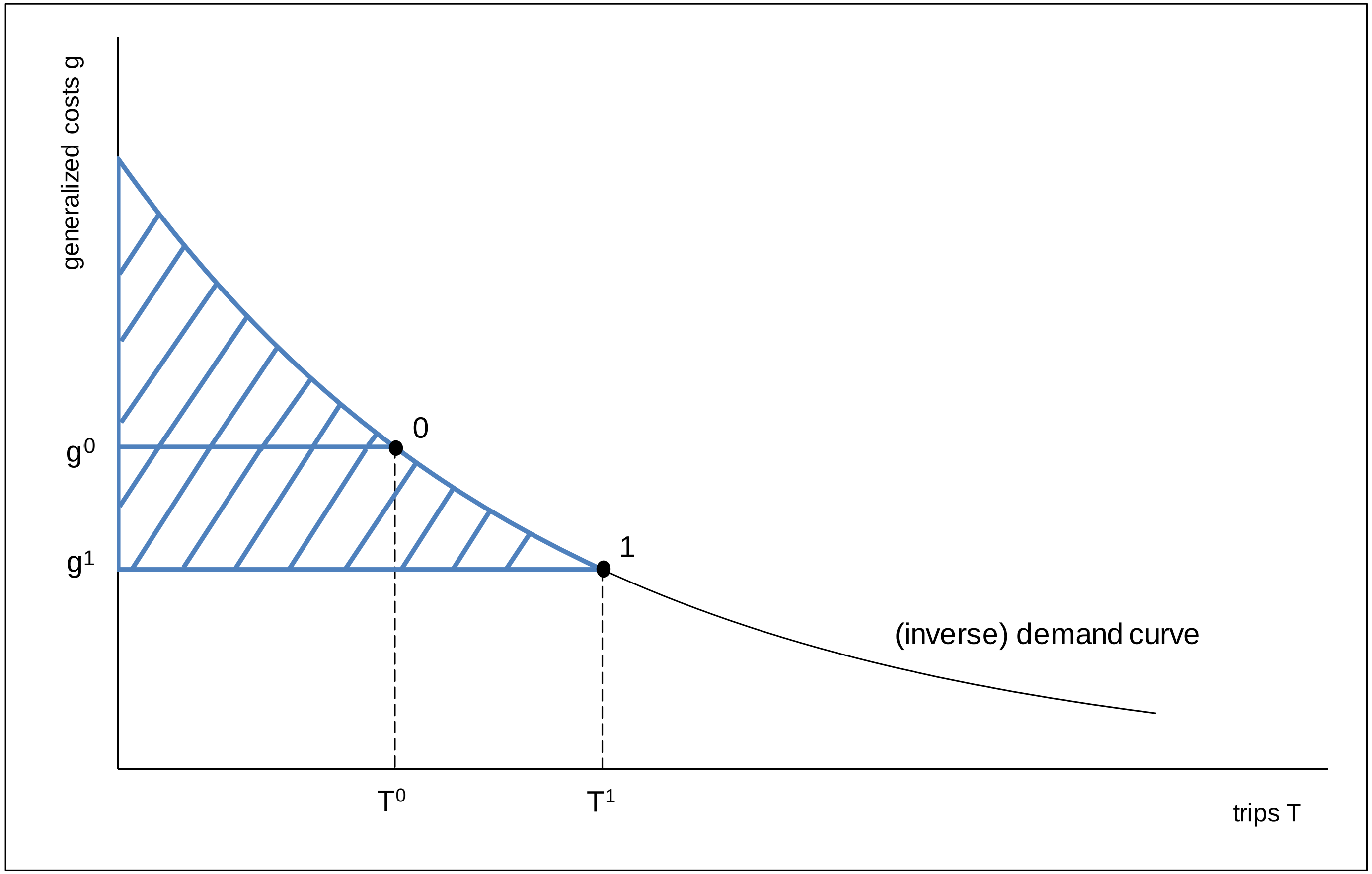
		Benefits						Costs
Overall Economic Impact	=	**change in transport user benefits**	+	change in system operating costs and revenues	+	change in costs of externalities	-	Investment costs



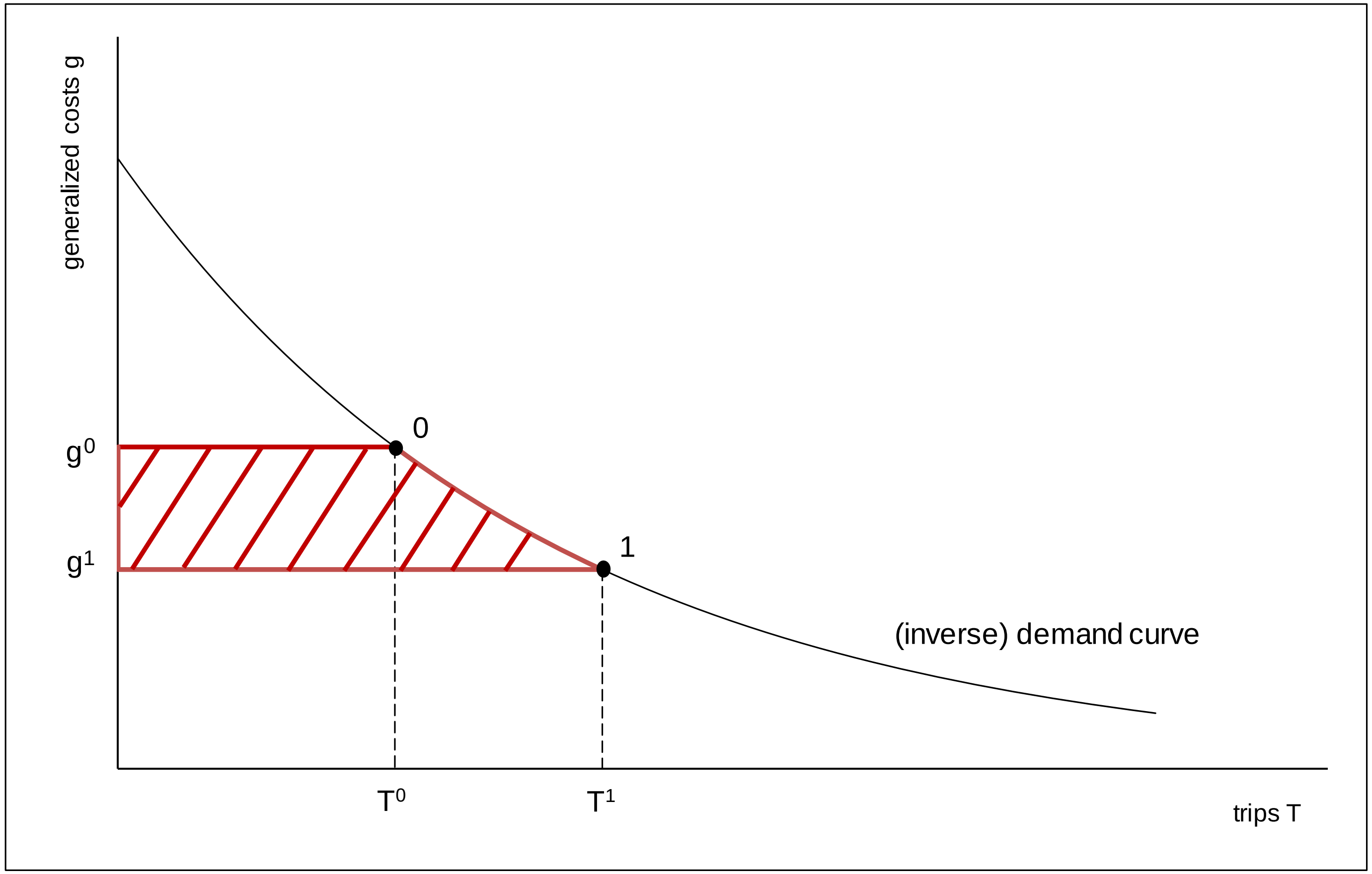
Consumer surplus – initial state



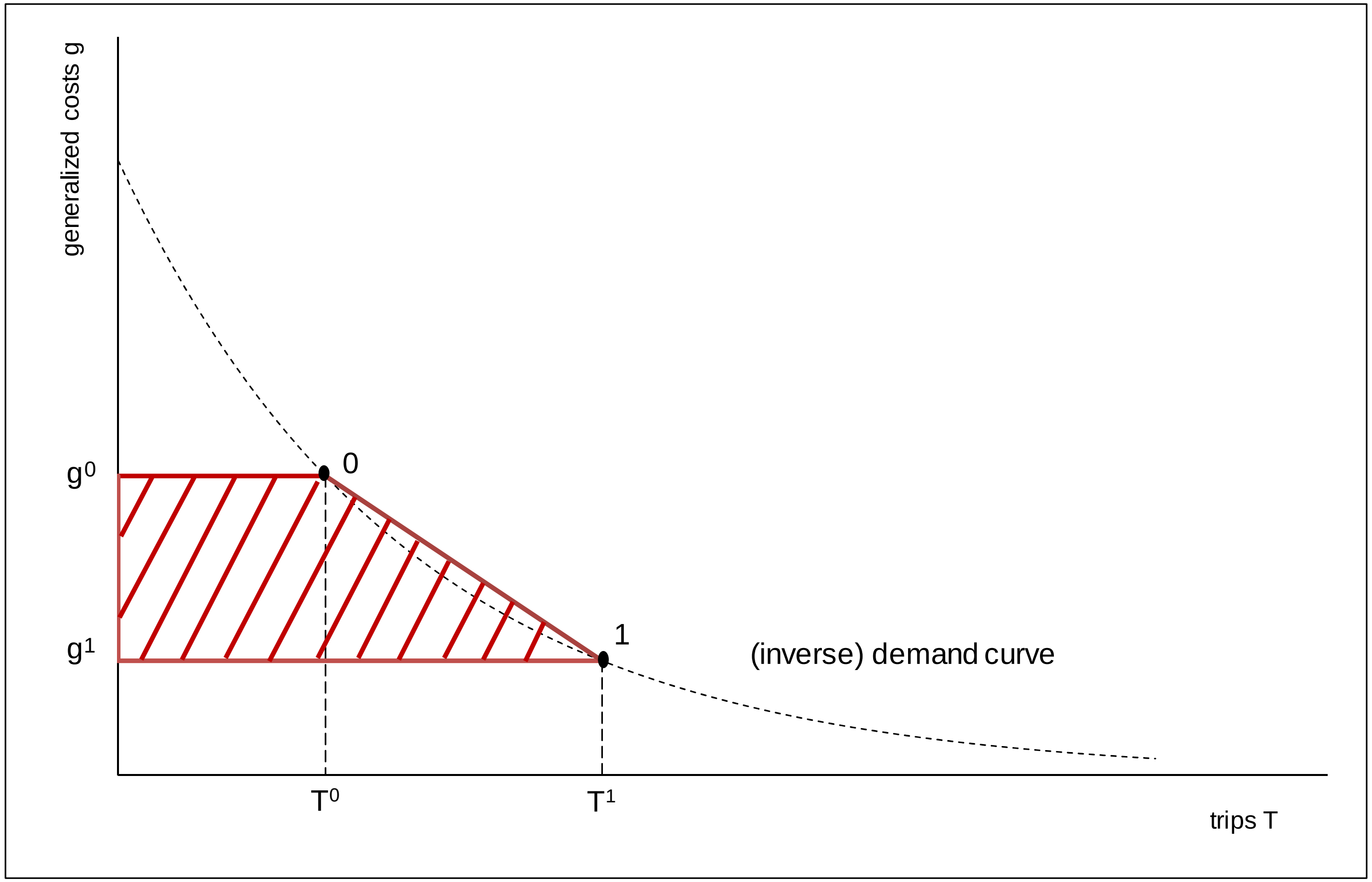
Consumer surplus – final state



Change in Consumer surplus



The Rule of the Half



Aim of the Study

- combine travel demand and benefits calculation for constrained travel demand models
- On the base of a trebly constrained model (EVA model)
- Change in consumer surplus as a benefit measure
- Compute a mathematical exact integration of the travel demand function

EVA logit model

EVA model

$$T_{ijk} = f\left(g_{ijk}\right) \cdot a_i \cdot b_j \cdot c_k$$

$$\left.\begin{array}{l} \sum_j \sum_k T_{ijk} = O_i \\ \sum_i \sum_k T_{ijk} = D_j \\ \sum_i \sum_j T_{ijk} = M_k \end{array}\right\} \text{constraints}$$

- Consideration of different sets of constraints
- Utility maximization?

Logit model

$$T_{ijk} = P_{ijk} \cdot T = \frac{e^{V_{ijk}}}{\sum_{i'} \sum_{j'} \sum_{k'} e^{V_{i'j'k'}}} \cdot T$$

- Consideration of only one set of constraints
- Random utility maximization

EVA logit model

- Definition of a specific utility function
- Consideration of shadow prices

$$\boxed{V_{ijk} = -g_{ijk} + \theta_i + \tau_j + \rho_k \quad \forall i, j, k}$$

- The EVA logit model is then:

$$T_{ijk} = P_{ijk} \cdot T = \frac{e^{\left(-g_{ijk} + \theta_i + \tau_j + \rho_k\right)}}{\sum_{i'} \sum_{j'} \sum_{k'} e^{\left(-g_{i'j'k'} + \theta_{i'} + \tau_{j'} + \rho_{k'}\right)}} \cdot T$$

$$\sum_j \sum_k T_{ijk} = O_i$$

$$\sum_i \sum_k T_{ijk} = D_j$$

$$\sum_i \sum_j T_{ijk} = M_k$$

Change in consumer surplus

- The change in consumer surplus is mandatory for defining welfare impacts
- The mathematical integration of the demand function is necessary
- Use either the single integration of each alternative or the antiderivative
- The antiderivative of the logit model is known (“logsum term”)

- The change in consumer surplus is then (“logsum difference”):

$$\boxed{E(\Delta CS) = \left[\ln\left(\sum_i \sum_j \sum_k e^{V_{ijk}^{1}} \right) - \ln\left(\sum_i \sum_j \sum_k e^{V_{ijk}^{0}} \right) \right] \cdot T}$$

Change in consumer surplus

- However: logsum difference fails for constrained models
- Reason: shadow prices for satisfying constraints
- Only the real benefit caused by changes in generalized costs is targeted
- A detailed analysis of the integral of the demand function is necessary
- Segmentation of the integral according to single variables

Change in consumer surplus

- Logsum difference approach with shadow prices can be partitioned into:

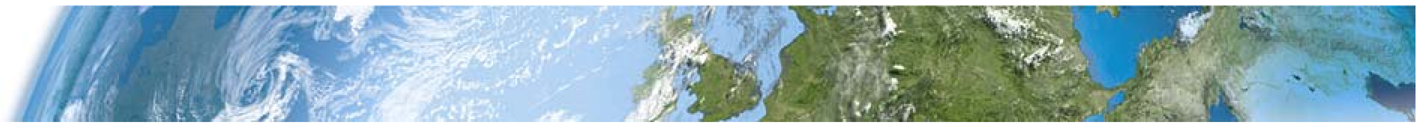
$$E\left(\Delta CS_r^*\right) = -\int_{g^0}^{g^1} \sum_i \sum_j \sum_k P_{ijk}\, dg_{ijk} + \int_{\theta^0}^{\theta^1} \sum_i \sum_j \sum_k P_{ijk}\, d\theta_i + \int_{\tau^0}^{\tau^1} \sum_i \sum_j \sum_k P_{ijk}\, d\tau_j + \int_{\rho^0}^{\rho^1} \sum_i \sum_j \sum_k P_{ijk}\, d\rho_k$$

- Possible simplifications:

$$E\left(\Delta CS_r\right) = E\left(\Delta CS_r^*\right) - \int_{\theta^0}^{\theta^1} \sum_i P_i\, d\theta_i - \int_{\tau^0}^{\tau^1} \sum_j P_j\, d\tau_j$$

- In the case of inelastic constraints it is:

$$E\left(\Delta CS_r^*\right) = 0, \quad P_i = \frac{O_i}{T} = \text{constant}, \quad P_j = \frac{D_j}{T} = \text{constant}$$



Change in consumer surplus

- It follows:

$$E\left(\Delta CS_r\right) = \sum_i \frac{O_i}{T} \cdot \left(\theta_i^0 - \theta_i^1\right) + \sum_i \frac{D_j}{T} \cdot \left(\tau_i^0 - \tau_i^1\right)$$

- Overall change in consumer surplus:

$$E\left(\Delta CS\right) = \sum_i O_i \cdot \left(\theta_i^0 - \theta_i^1\right) + \sum_i D_j \cdot \left(\tau_i^0 - \tau_i^1\right)$$

- The result is valid for inelastic constraints and constant shadow prices for modes
- Solutions for all different kinds of constraints have been derived

Example

- Synthetic example:
  - five travel zones
  - inelastic origin and destination constraints
  - one mode
  - travel time and travel cost
  - value of time of 10 Euro/h (0.1667 Euro/min)

initial state

generalized costs [Euro]

gij	1	2	3	4	5
1	4,08	3,25	6,25	5,50	3,25
2	4,08	4,08	4,78	4,58	5,00
3	6,25	4,37	4,08	4,00	4,00
4	5,50	4,58	4,42	4,08	4,08
5	4,50	6,33	4,00	3,25	3,67

final state

generalized costs [Euro]

gij	1	2	3	4	5
1	4,08	3,25	6,25	3,25	3,25
2	4,08	4,08	4,78	4,58	5,00
3	6,25	4,37	4,08	4,00	4,00
4	3,25	4,58	4,42	4,08	4,08
5	4,50	6,33	4,00	3,25	3,67

Example – EVA logit model

- Calculation of trips by the EVA logit model – **initial state**

Tij	**1**	**2**	**3**	**4**	**5**		**Oi-resulting**	**Oi-given**	**θi**
1	3,03	32,35	1,93	1,38	11,31		50,00	50,00	2,611
2	9,81	45,52	27,14	11,17	6,36		100,00	100,00	3,785
3	0,44	13,46	21,46	7,86	6,79		50,00	50,00	2,850
4	2,30	26,68	37,85	17,80	15,37		100,00	100,00	3,751
5	9,42	6,99	86,62	61,79	35,18		200,00	200,00	4,162
						500,00	**500,00**	**500,00**	
Dj-resulting	25,00	125,00	175,00	100,00	75,00	**500,00**			
Dj-given	25,00	125,00	175,00	100,00	75,00	**500,00**			
τj	2,581	4,116	4,299	3,211	3,065				

Example – EVA logit model

- Calculation of trips by the EVA logit model – **initial state**

Tij	1	2	3	4	5		Oi-resulting	Oi-given	θi
1	3,03	32,35	1,93	1,38	11,31		50,00	50,00	2,611
2	9,81	45,52	27,14	11,17	6,36		100,00	100,00	3,785
3	0,44	13,46	21,46	7,86	6,79		50,00	50,00	2,850
4	2,30	26,68	37,85	17,80	15,37		100,00	100,00	3,751
5	9,42	6,99	86,62	61,79	35,18		200,00	200,00	4,162
						500,00	**500,00**	**500,00**	
Dj-resulting	25,00	125,00	175,00	100,00	75,00	**500,00**			
Dj-given	25,00	125,00	175,00	100,00	75,00	**500,00**			
τj	2,581	4,116	4,299	3,211	3,065				

$$T_{11} = P_{11} \cdot T$$

$$= \frac{e^{(-4{,}08\,\text{Euro}+2{,}611\,\text{Euro}+2{,}581\,\text{Euro})_{11}}}{\sum_{i'}\sum_{j'} e^{(-g_{i'j'}+\theta_{i'}+\tau_{j'})}} \cdot 500 \text{ trips}$$

$$= 3{,}03 \text{ trips}$$

Example – EVA logit model

- Calculation of trips by the EVA logit model – **final state**

Tij	**1**	**2**	**3**	**4**	**5**		**Oi-resulting**	**Oi-given**	**θi**
1	1,45	27,93	1,57	9,62	9,43		50,00	50,00	2,386
2	5,90	49,43	27,67	10,33	6,67		100,00	100,00	3,790
3	0,26	14,29	21,39	7,10	6,96		50,00	50,00	2,833
4	11,59	25,58	34,07	14,53	14,23		100,00	100,00	3,632
5	5,80	7,77	90,30	58,41	37,72		200,00	200,00	4,190
						500,00	**500,00**	**500,00**	
Dj-resulting	25,00	125,00	175,00	100,00	75,00	**500,00**			
Dj-given	25,00	125,00	175,00	100,00	75,00	**500,00**			
τj	2,068	4,194	4,313	3,128	3,107				

Example – change in consumer surplus

- Change in consumer surplus calculated by the adjusted logsum difference

total number of trips originating at zone i		difference of origin shadow prices			ΔCS_i [Euro]
O_1	50	$\theta^O{}_1-\theta^M{}_1$	0,224		11,22
O_2	100	$\theta^O{}_2-\theta^M{}_2$	-0,005		-0,49
O_3	50	$\theta^O{}_3-\theta^M{}_3$	0,018		0,88
O_4	100	$\theta^O{}_4-\theta^M{}_4$	0,120		11,96
O_5	200	$\theta^O{}_5-\theta^M{}_5$	-0,027		-5,45
				Σ(i)	18,11

total number of trips attracted to zone j		difference of destination shadow prices			ΔCS_j [Euro]
D_1	25	$\tau^O{}_1-\tau^M{}_1$	0,513		12,82
D_2	125	$\tau^O{}_2-\tau^M{}_2$	-0,078		-9,69
D_3	175	$\tau^O{}_3-\tau^M{}_3$	-0,014		-2,52
D_4	100	$\tau^O{}_4-\tau^M{}_4$	0,083		8,34
D_5	75	$\tau^O{}_5-\tau^M{}_5$	-0,042		-3,19
				Σ(j)	5,76

Σ(i,j)	**23,87**	Euro

$$E(\Delta CS) = \sum_i O_i \cdot \left(\theta_i^1 - \theta_i^0\right) + \sum_i D_j \cdot \left(\tau_i^1 - \tau_i^0\right)$$

Example – change in consumer surplus

- Change in consumer surplus calculated by the Rule of a Half

Δgij	**1**	**2**	**3**	**4**	**5**
1	0,00	0,00	0,00	2,25	0,00
2	0,00	0,00	0,00	0,00	0,00
3	0,00	0,00	0,00	0,00	0,00
4	2,25	0,00	0,00	0,00	0,00
5	0,00	0,00	0,00	0,00	0,00

ΣTij/2	**1**	**2**	**3**	**4**	**5**
1	2,24	30,14	1,75	5,50	10,37
2	7,85	47,47	27,41	10,75	6,51
3	0,35	13,87	21,42	7,48	6,87
4	6,94	26,13	35,96	16,17	14,80
5	7,61	7,38	88,46	60,10	36,45

ΔCS	**1**	**2**	**3**	**4**	**5**
1	0,00	0,00	0,00	12,38	0,00
2	0,00	0,00	0,00	0,00	0,00
3	0,00	0,00	0,00	0,00	0,00
4	15,62	0,00	0,00	0,00	0,00
5	0,00	0,00	0,00	0,00	0,00

28,00 Euro

$$E(\Delta CS) = \sum_i \sum_j \frac{1}{2} \cdot \left(g_{ij}^0 - g_{ij}^1\right) \cdot \left(T_{ij}^0 + T_{ij}^1\right)$$

- Benefit is overestimated by 17%

Conclusion

- Multiple-constrained travel demand models are expressible in terms of a logit model
- Derivation of the “adjusted logsum difference” as a mathematical exact integral of the demand function of constrained models
- Applicable for all constrained models
- Allows measuring benefits caused by changing costs and changing attractions
- The approach is easy to use and to implement in existing models

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